Student's Name: ____________________

Student Number: | | | | | | | | | |

Teacher's Name: ____________________

ABBOTSLEIGH

2020

HIGHER SCHOOL CERTIFICATE

Assessment Task 4

Advanced Mathematics

General Instructions

- Reading time – 10 minutes.
- Working time – 3 hours
- Write using black pen.
- **NESA approved** calculators may be used.
- **NESA approved** reference sheet is provided.
- All necessary working should be shown in every question to gain full marks.
- Make sure your Student Number is on the front cover of each section.
- Answer the Multiple-Choice questions on the answer sheet provided.
- In Questions 11 - 16, show relevant mathematical reasoning and/ or calculations

Total marks – 100

- Attempt Sections I and II

Section I Pages 3 - 8

10 marks

- Attempt Questions 1–10.
- Allow about 15 minutes for this section.

Section II Pages 9 - 44

90 marks

- Attempt Questions 11– 16.
- Allow about 2 hrs and 45 minutes for this section

Outcomes to be assessed:

Mathematics

Preliminary:

A student

MA11-1 uses algebraic and graphical techniques to solve, and where appropriate, compare alternative solutions to problems

MA11-2 uses the concepts of functions and relations to model, analyse and solve practical problems

MA11-3 uses the concepts and techniques of trigonometry in the solution of equations and problems involving geometric shapes

MA11-4 uses the concepts and techniques of periodic functions in the solutions of trigonometric equations or proof of trigonometric identities

MA11-5 interprets the meaning of the derivative, determines the derivative of functions and applies these to solve simple practical problems

MA11-6 manipulates, solves expressions using the logarithmic & index laws, uses logarithms, exponential functions to solve practical problems

MA11-7 uses concepts and techniques from probability to present and interpret data and solve problems in a variety of contexts, including the use of probability distributions

MA11-8 uses appropriate technology to investigate, organise, model and interpret information in a range of contexts

MA11-9 provides reasoning to support conclusions which are appropriate to the context

HSC:

A student

MA12-1 uses detailed algebraic and graphical techniques to critically construct, model and evaluate arguments in a range of familiar and unfamiliar contexts

MA12-2 models and solves problems and makes informed decisions using mathematical reasoning and techniques

MA12-3 applies calculus techniques to model and solve problems

MA12-4 applies the concepts and techniques of arithmetic and geometric sequences and series in the solution of problems

MA12-5 applies the concepts and techniques of periodic functions in the solution of problems involving trigonometric graphs

MA12-6 applies appropriate differentiation methods to solve problems

MA12-7 applies the concepts and techniques of indefinite and definite integrals in the solution of problems

MA12-8 solves problems using appropriate statistical processes

MA12-9 chooses and uses appropriate technology effectively in a range of contexts, models and applies critical thinking to recognise appropriate times for such use

MA12-10 constructs arguments to prove and justify results and provides reasoning to support conclusions which are appropriate to the context

SECTION I

10 marks

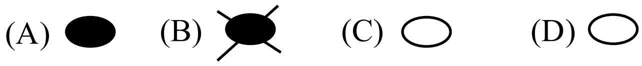
Attempt Questions 1 – 10

Use the multiple-choice answer sheet

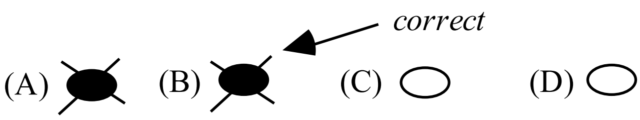
Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample $2 + 4 =$ (A) 2 (B) 6 (C) 8 (D) 9

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.



If you change your mind and have crossed out what you consider to be the correct answer, then indicate this by writing the word *correct* and drawing an arrow as follows.



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1. What are the solutions to the equation $\sin x = \dfrac{\sqrt{3}}{2}$, for $0 \le x \le 2\pi$?

A. $\dfrac{\pi}{6}, \dfrac{5\pi}{6}$

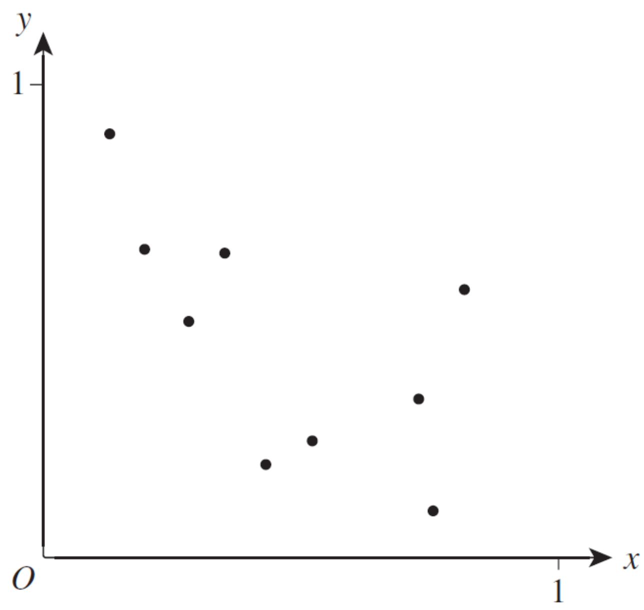
B. $\dfrac{\pi}{3}, \dfrac{2\pi}{3}$

C. $\dfrac{\pi}{4}, \dfrac{3\pi}{4}$

D. $\dfrac{\pi}{2}, \dfrac{3\pi}{2}$

2. A scatterplot relates the quantities x and y.

How could you describe the correlation between those quantities?



A. A moderate negative correlation

B. A moderate positive correlation

C. A weak positive correlation.

D. A strong negative correlation

3. For what values of x is the curve $f(x) = 2x^3 + x^2$ concave down?

A. $x < -\frac{1}{6}$

B. $x > 6$

C. $x < -6$

D. $x > -\frac{1}{6}$

4. What is the period for the curve $y = -3\cos\left(2x - \frac{\pi}{4}\right)$?

A. 3

B. π

C. 2π

D. -3

5. Which one of the following is the set of all solutions to $2x^2 - 5x + 2 \geq 0$?

A. $\left[\frac{1}{2}, 2\right]$

B. $\left(\frac{1}{2}, 2\right)$

C. $\left(-\infty, \frac{1}{2}\right) \cup (2, \infty)$

D. $\left(-\infty, \frac{1}{2}\right] \cup [2, \infty)$

6. What is the value of $f'(x)$ if $f(x) = 3x^4(4-x)^3$?

A. $3x^3(4-x)^2(7x-16)$

B. $3x^3(4-x)^2(16-7x)$

C. $3x^3(4-x)^3(16-7x)$

D. $3x^3(4-x)^3(7x-16)$

7. The graph of $y = f(x)$ has a stationary point at $(2, -3)$.

Consequently, which of the following is a stationary point of $y = -f\left(\frac{x}{2}\right) - 5$?

A. $(4, 2)$

B. $(4, -2)$

C. $(1, 2)$

D. $(1, -2)$

8. For the series $2\pi, \pi, \frac{\pi}{2}, ...$, calculate the exact value of the sum of the first 6 terms.

A. $\frac{63\pi}{16}$

B. $\frac{7\pi}{2}$

C. $\frac{977\pi}{256}$

D. $\frac{63\pi}{64}$

9. Consider the region bounded by the x-axis, the y-axis, the line with equation $y = 3$ and the curve with equation $y = \ln(x - 1)$. The exact value of the area of this region is:

A. $e^{-3} - 1$

B. $e^{3} + 2$

C. $3e^{2}$

D. $3e^{3} - e^{-3} + 2$

10. A lie detector was used to indicate the guilt or innocence of 200 suspects.

	Accurate	Not accurate	Total
True statements	95	10	105
False statements	70	25	95
Total	165	35	200

What is the probability a person selected at random, with an accurate test, made a true statement?

A. $\frac{95}{105}$

B. $\frac{95}{200}$

C. $\frac{95}{165}$

D. $\frac{165}{200}$

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Section II **Student Number:**

90 marks

Allow about 2 hours and 45 minutes for this section

Answer each question in the spaces provided.

Your responses should include relevant mathematical reasoning and/or calculations.

Extra writing space is provided at the end of each question. If you use this space, clearly indicate which question you are answering.

Question 11 (15 marks) **Marks**

(a) Find a primitive of $\frac{1}{5x+1}$. **2**

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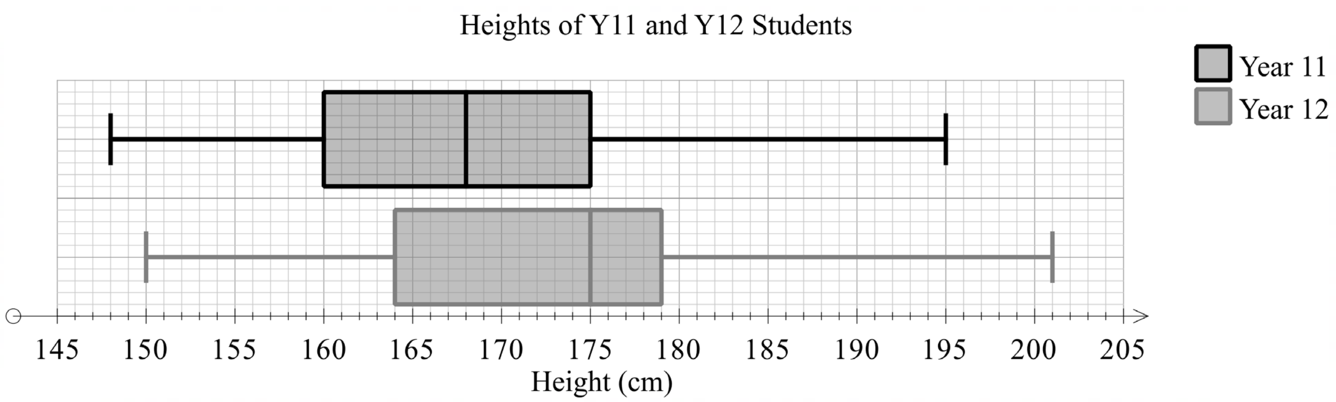
(b) The boxplot shows the heights of students in Year 11 and Year 12 at a school.



(i) What percentage of students in Year 11 have a height below 160 cm? **1**

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(ii) The number of students taller than 175 cm is coincidentally the same for both Year 11 and Year 12. If Year 11 has a total of 140 students, how many students are in Year 12? **1**

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Question 11 continues on page 10

(c) Find the common ratio of a geometric series with a first term of $\sqrt{2}$ and a limiting sum of $\frac{3\sqrt{2}}{2}$. **2**

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(d) Find $\int \frac{8x^3 - 3}{x^2}\, dx$ **2**

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Question 11 continues on page 11

(e) A curve with the equation $y = f(x)$, has $\frac{dy}{dx} = x^3 + 2x - 7$.

(i) Find $\frac{d^2y}{dx^2}$ **1**

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(ii) Show that $\frac{d^2y}{dx^2} \geq 2$ for all values of x. **2**

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(iii) The point $P(2, 4)$ lies on the curve. Find y in terms of x. **2**

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Question 11(e) continues on page 12

(iv) Find an equation for the normal to the curve at P, in the form $ax + by + c = 0$, where a, b and c are integers. **2**

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End of Question 11

Section II Extra writing space

If you use this space, clearly indicate which question you are answering.

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Question 12 (15 marks) **Student Number:** | | | | | | | | | |

(a) Let $h(x) = (x-2)(x^2+1)$.

(i) Find where the graph of $y = h(x)$ cuts the x-axis and y-axis. **2**

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(ii) Find the coordinates of the stationary points on the curve with the equation $y = h(x)$ and determine their nature. **5**

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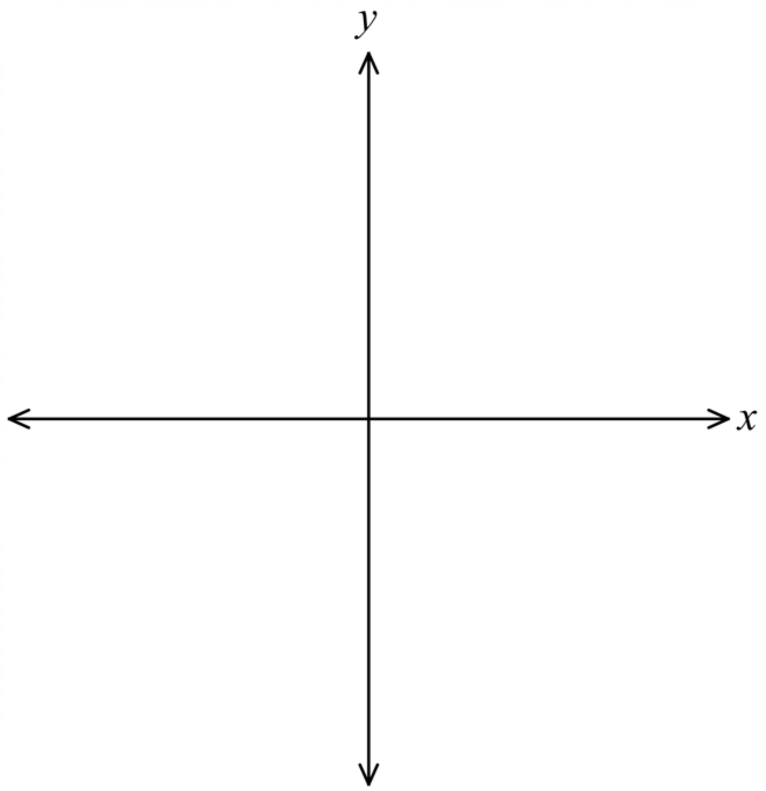
Question 12 continues on page 16

Question 12 (continued)

(iii) Sketch the graph of $y = h(x)$ showing stationary points and axes intercepts. **2**



(b) If $\tan\theta = \frac{4}{5}$, and θ is acute, find the exact value of $\sin\theta$. **2**

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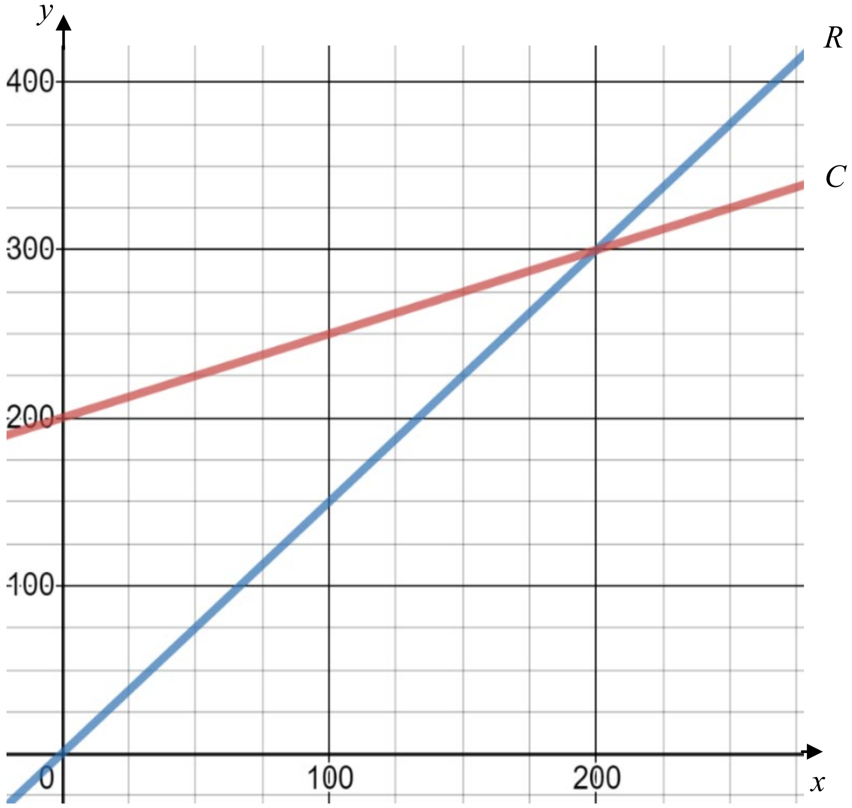
Question 12 continues on page 17

(c) Terry is starting a small business making face masks.

Technology was used to draw straight-line graphs to represent the cost C, of Terry making face masks and the revenue, R from him selling them. The x-axis displays the number of face masks and the y-axis displays the cost/revenue in dollars.



(i) How many face masks must Terry sell to break even? **1**

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Question 12(c) continues on page 18

(ii) By first forming equations for cost C, and revenue R, determine how many face masks need to be sold to earn Terry a profit of \$1500. **3**

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End of Question 12

Section II Extra writing space

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Question 13 (15 marks) **Student Number:** | | | | | | | | | |

(a) Find the exact value of $\int_0^{\frac{\pi}{6}} \sec^2 2x \, dx$ **2**

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(b) Differentiate with respect to x.

(i) $y = \ln\left(3x^2 + 1\right)$ **1**

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(ii) $y = \frac{\sin x}{x^2}$ **2**

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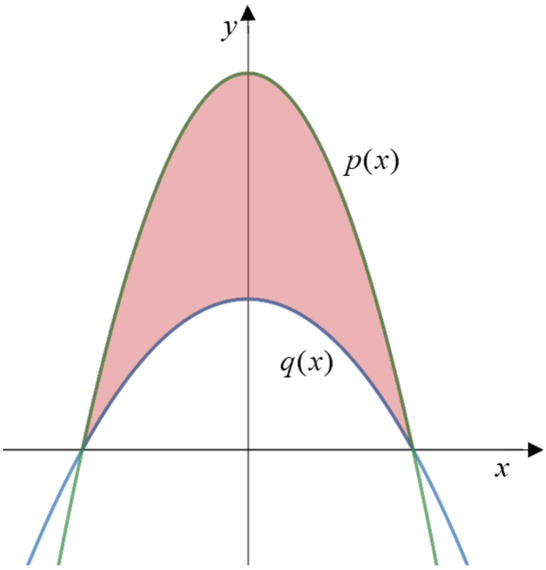
Question 13 continues on page 22

Question 13 (continued)

(c) Tess is creating a logo from the region intersecting the curves:

$$p(x) = (5-x)(5+x) \quad \text{and} \quad q(x) = \frac{2}{5}(5-x)(5+x).$$



(i) Show that the area A, of the shaded region is given by the expression **3**

$$A = \frac{6}{5}\int_0^5 25 - x^2 \, dx.$$

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Question 13(c) continues on page 23

(ii) Hence, or otherwise, find the area of the shaded region. **2**

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(d) For events A and B from a sample space, $P(A \mid B) = \frac{3}{4}$ and $P(B) = \frac{1}{7}$. **1**

Calculate $P(A \cap B)$.

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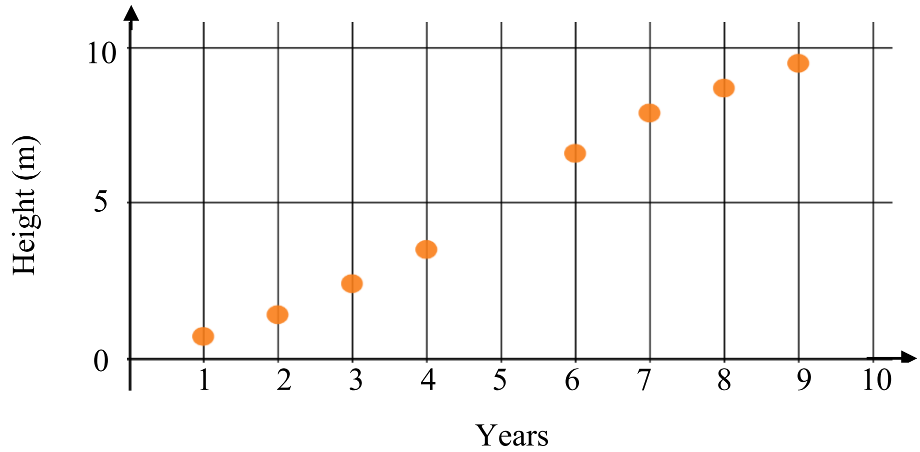
Question 13 continues on page 24

(e) Charlotte is an agricultural scientist studying the growth of a particular tree over several years. The data she recorded is shown in the table and graph below.

Years since planting, t	1	2	3	4	6	7	8	9
Height of tree, H metres	0.7	1.4	2.4	3.5	6.6	7.9	8.7	9.5



(i) What is the correlation coefficient for this data (correct to 4 decimal places)? **1**

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(ii) Find the equation of the least-squares line of best fit in terms of years (t) and height (H). Answer using values A and B correct to 2 decimal places, where $H = A + Bt$. **1**

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(iii) Use the equation to approximately determine how many years it will take for the tree to reach a height of 20 metres. Answer correct to 1 decimal place. **1**

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(iv) What is the limitation of this model? **1**

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End of Question 13

Section II Extra writing space

If you use this space, clearly indicate which question you are answering.

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Question 14 (15 marks) **Student Number:** | | | | | | | | | |

(a) The probability that Chloe gets a concert booking with her band on any given weekend is 65%. What is the probability that she gets at least one booking over two consecutive weekends? **2**

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(b) A circle is given by the equation $x^2 + y^2 - 4x + 6y = 12$.

Find the centre and radius of this circle. **3**

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Question 14 continues on page 28

(c) The score, X, for a biased spinner is given by the probability distribution:

x	2	4	6
$P(X = x)$	$\frac{1}{12}$	$\frac{2}{3}$	p

By finding the value of p , calculate the expected value and the variance of X. **3**

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Question 14 continues on page 29

(d) The displacement of a particle is given by $x = t^2 - 4\log_e(t-1) + 5$,

where x is in metres, t is in seconds and $t > 1$.

(i) Find the exact displacement of the particle when $t = 4$. **1**

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(ii) Find an expression for the particle's velocity and hence find when the particle comes to rest. **2**

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(iii) Show that the acceleration remains positive for $t > 1$. **2**

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Question 14(d) continues on page 30

(iv) Find the exact distance travelled by the particle between the times the particle comes to rest and $t = 4$. **2**

End of Question 14

Section II Extra writing space

If you use this space, clearly indicate which question you are answering.

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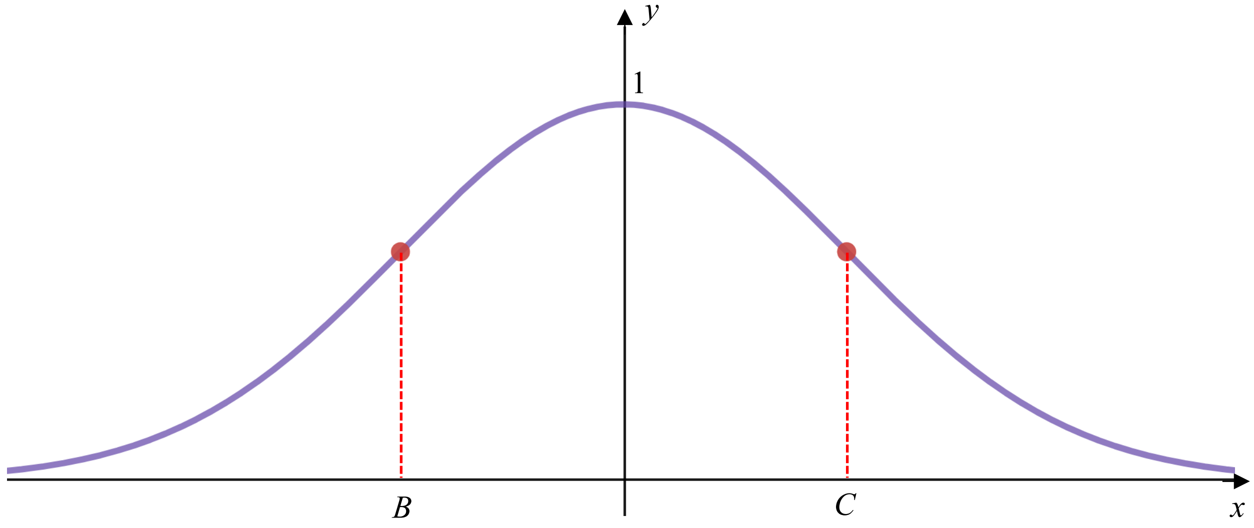
Question 15 (15 marks) **Student Number:**

(a) Isabelle is exploring the curve of the even function shown below, $y = e^{-x^2}$.
She knows there is a single stationary point shown at (0, 1) and two points of inflection are shown with x-values of B and C.



(i) Show that $\dfrac{d^2y}{dx^2} = 4e^{-x^2}\left(x^2 - \dfrac{1}{2}\right)$ **2**

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Question 15(a) continues on page 34

Question 15(a) (continued)

(ii) Hence find the coordinates of the two points of inflection. **2**

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Isabelle wants to find the area of the region under the curve $y = e^{-x^2}$ bounded by the x-axis and the two inflection points, that is, $\int_B^C e^{-x^2}\, dx$.

(iii) Explain how using a formula given on the Reference Sheet is unable to help provide an answer. **1**

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Question 15(a) continues on page 35

(iv) Isabelle decides to approximate the area using the Trapezoidal Rule.

Show how she determined $\int_B^C e^{-x^2}\,dx \approx \frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2e}}$ using three function values. **3**

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(v) Explain why Isabelle correctly knows $\int_B^C e^{-x^2}\,dx > \frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2e}}$. **1**

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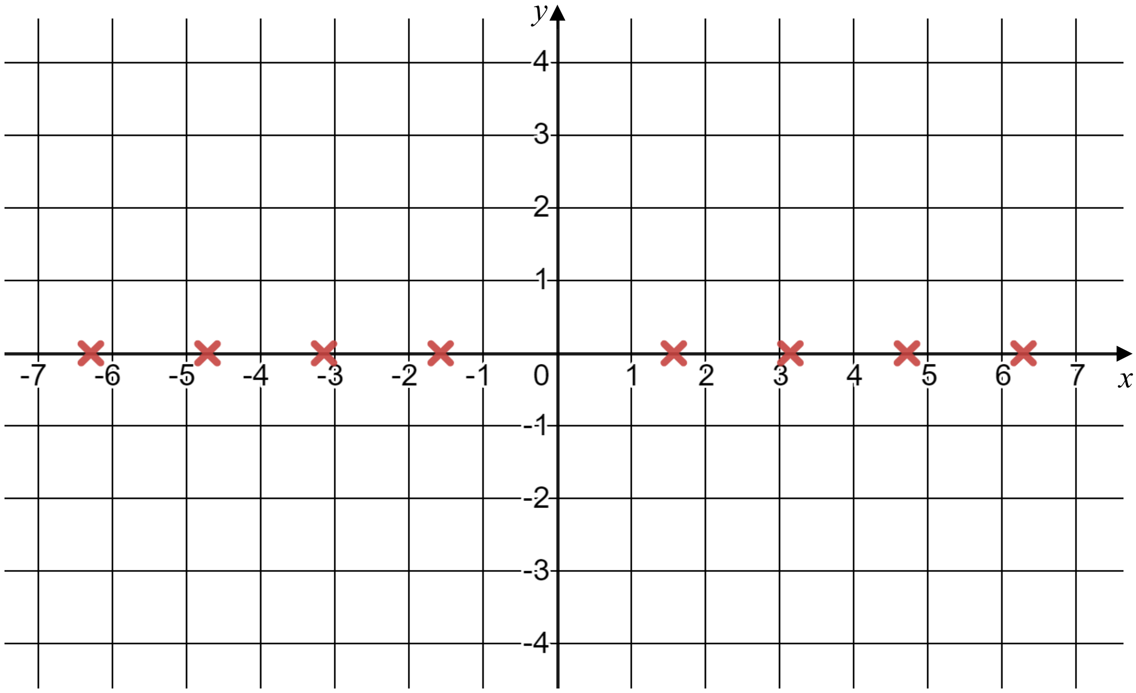
Question 15 continues on page 36

Question 15 (continued)

(b) Annie was preparing to determine how many solutions there are to the equation:

$$3\sin x = \frac{x^2}{5} - 3.$$

She plotted multiples of $\frac{\pi}{2}$ on the x-axis of the number plane below, shown by the crosses, to help. Draw graphs on this number plane to solve Annie's problem. **3**



The number of solutions: _______________

Question 15 continues on page 37

(c) Show that $\sqrt{\dfrac{\sec\theta - 1}{\sec\theta + 1}} = \operatorname{cosec}\theta - \cot\theta$ **3**

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End of Question 15

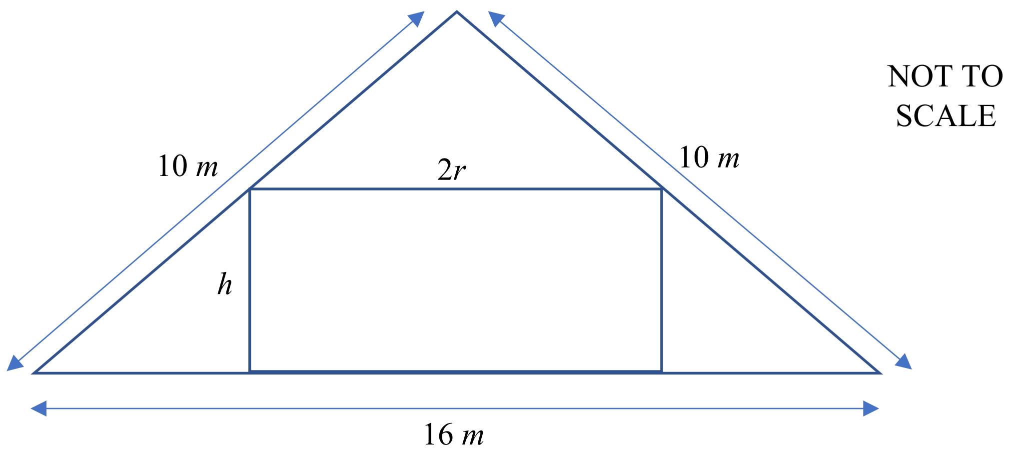
Section II Extra writing space

If you use this space, clearly indicate which question you are answering.

Question 16 (15 marks) **Student Number:** | | | | | | | | | |

(a) In some rural areas hot water tanks are installed in the roofs of houses. The diagram below shows a cross-section of a cylindrical tank in a roof. The cylindrical tank snugly fits exactly into the roof with diameter $2r$ metres and height h metres.

The cross-section of the roof is an isosceles triangle with dimensions show.



(i) Show that the height of the roof is 6 metres. **1**

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(ii) Hence show that $h = \frac{3}{4}(8 - r)$. **2**

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Question 16(a) continues on page 40

(iii) Show that the volume of the cylindrical tank can be expressed by **1**

$$V = \frac{3\pi}{4}\left(8r^2 - r^3\right).$$

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(iv) Find the value of r which gives the tank its greatest volume and calculate that volume, correct to the nearest litre. **4**

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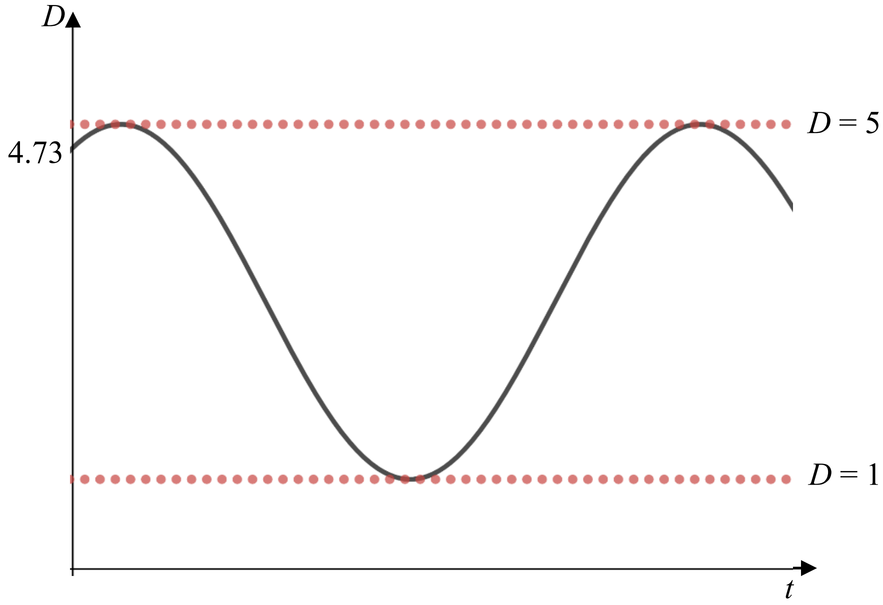
Question 16 continues on page 41

Question 16 (continued)

(b)



Sophie has developed an equation, drawn above, for the depth D, of a river near her home. The depth is modelled by the function:

$$D = a\sin\left(nt + \frac{\pi}{3}\right) + c$$

where D is measured in metres and t is the time in hours. The time between successive peaks in Sophie's model is exactly 12 hours.

(i) What is the value of the amplitude, a? **1**

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(ii) Find the value of c. **1**

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Question 16(b) continues on page 42

Question 16(b) (continued)

(iii) Find the value of n. **1**

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Sophie started recording the river depth when it was 4.73 metres and waited to cross it safely.

(iv) How long did she have to wait until it had a depth of 1 metre? **2**

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. **Question 16(b) continues on page 43**

Question 16(b) (continued)

(v) From her record, when was the greatest rate of drop in depth and what was that rate at this time? Answer correct to 2 decimal places **2**

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END OF PAPER

Section II Extra writing space

If you use this space, clearly indicate which question you are answering.

Student's Name: SOLUTIONS

Student Number:

Teacher's Name:

ABBOTSLEIGH

2020

HIGHER SCHOOL CERTIFICATE
Assessment Task 4

Advanced Mathematics

General Instructions

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- Working time – 3 hours
- Write using black pen.
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- **NESA approved** reference sheet is provided.
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Total marks – 100

- Attempt Sections I and II

Section I Pages 3 - 8

10 marks

- Attempt Questions 1–10.
- Allow about 15 minutes for this section.

Section II Pages 9 - 44

90 marks

- Attempt Questions 11– 16.
- Allow about 2 hrs and 45 minutes for this section

10 marks
Attempt Questions 1 – 10
Use the multiple-choice answer sheet
Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample $2 + 4 =$ (A) 2 (B) 6 (C) 8 (D) 9

(A) ○ (B) ● (C) ○ (D) ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

(A) ● (B) ⊗ (C) ○ (D) ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate this by writing the word *correct* and drawing an arrow as follows.

correct

(A) ⊗ (B) ⊗ (C) ○ (D) ○

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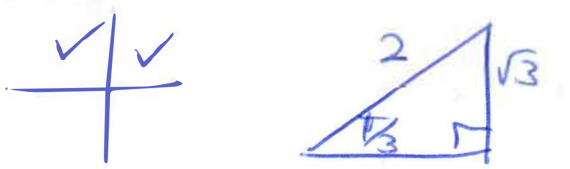
1. What are the solutions to the equation $\sin x = \dfrac{\sqrt{3}}{2}$, for $0 \le x \le 2\pi$?



A. $\dfrac{\pi}{6}, \dfrac{5\pi}{6}$

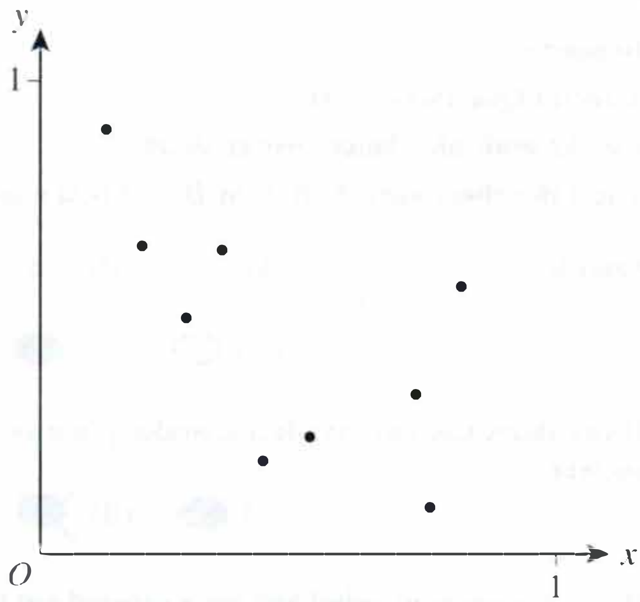
B. $\dfrac{\pi}{3}, \dfrac{2\pi}{3}$

C. $\dfrac{\pi}{4}, \dfrac{3\pi}{4}$

D. $\dfrac{\pi}{2}, \dfrac{3\pi}{2}$

2. A scatterplot relates the quantities x and y.

How could you describe the correlation between those quantities?



A. A moderate negative correlation

B. A moderate positive correlation

C. A weak positive correlation.

D. A strong negative correlation

3. For what values of x is the curve $f(x) = 2x^3 + x^2$ concave down?

A. $x < -\frac{1}{6}$

B. $x > 6$

C. $x < -6$

D. $x > -\frac{1}{6}$

$f'(x) = 6x^2 + 2x$

$f''(x) = 12x + 2 < 0$

$12x < -2$

$x < -\frac{1}{6}$

4. What is the period for the curve $y = -3\cos\left(2x - \frac{\pi}{4}\right)$?

A. 3

B. π

C. 2π

D. -3

$= -3\cos\left(2\left(x - \frac{\pi}{8}\right)\right)$

period $= \frac{2\pi}{2} = \pi$

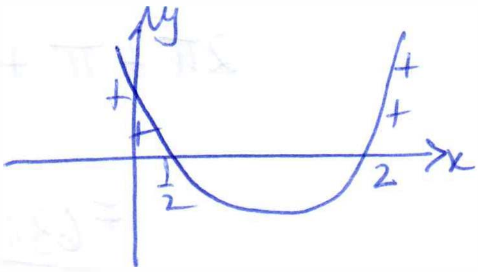
5. Which one of the following is the set of all solutions to $2x^2 - 5x + 2 \geq 0$?

A. $\left[\frac{1}{2}, 2\right]$

B. $\left(\frac{1}{2}, 2\right)$

C. $\left(-\infty, \frac{1}{2}\right) \cup (2, \infty)$

(D.) $\left(-\infty, \frac{1}{2}\right] \cup [2, \infty)$



6. What is the value of $f'(x)$ if $f(x) = 3x^4(4-x)^3$?

A. $3x^3(4-x)^2(7x-16)$

(B.) $3x^3(4-x)^2(16-7x)$

C. $3x^3(4-x)^3(16-7x)$

D. $3x^3(4-x)^3(7x-16)$

$f'(x) = 3x^4 \times -3(4-x)^2 + (4-x)^3 \times 12x^3$

$= -9x^4(4-x)^2 + 12x^3(4-x)^3$

$= 3x^3(4-x)^2[-3x + 4(4-x)]$

$= 3x^3(4-x)^2[-3x + 16 - 4x]$

$\therefore f'(x) = 3x^3(4-x)^2(16-7x)$

7. The graph of $y = f(x)$ has a stationary point at $(2, -3)$.

Consequently, which of the following is a stationary point of $y = -f\left(\frac{x}{2}\right) - 5$?

A. $(4,\ 2)$

(B.) $(4, -2)$

C. $(1,\ 2)$

D. $(1, -2)$

$(2, +3)$

$\times 2$

$(4, 3)$

-5

$(4, -2)$

8. For the series $2\pi, \pi, \frac{\pi}{2}, \ldots\ldots$, calculate the exact value of the sum of the first 6 terms.

A. $\frac{63\pi}{16}$

B. $\frac{7\pi}{2}$

C. $\frac{977\pi}{256}$

D. $\frac{63\pi}{64}$

$2\pi + \pi + \frac{\pi}{2} + \frac{\pi}{4} + \frac{\pi}{8} + \frac{\pi}{16}$

$= \frac{63\pi}{16}$

or $a = 2\pi$ $r = \frac{1}{2}$ $n = 6$

$S_6 = \frac{2\pi\left(\frac{1}{2^6} - 1\right)}{\left(\frac{1}{2} - 1\right)} = \frac{63\pi}{16}$.

9. Consider the region bounded by the x-axis, the y-axis, the line with equation $y = 3$ and the curve with equation $y = \ln(x - 1)$. The exact value of the area of this region is:

A. $e^{-3} - 1$

correct → B. $e^3 + 2$

C. $3e^2$

D. $3e^3 - e^{-3} + 2$

$y = \log_e(x-1)$

$e^y + 1 = x$

$A = \int_0^3 e^y + 1 \, dy$

$= \left[e^y + y\right]_0^3$

$= e^3 + 3 - (e^0 + 0)$

$= e^3 + 3 - 1$

$\therefore A = e^3 + 2$

10. A lie detector was used to indicate the guilt or innocence of 200 suspects.

	Accurate	Not accurate	Total
True statements	95	10	105
False statements	70	25	95
Total	165	35	200

What is the probability a person selected at random, with an accurate test, made a true statement?

$\frac{95}{165}$

A. $\frac{95}{105}$

B. $\frac{95}{200}$

C. $\frac{95}{165}$

D. $\frac{165}{200}$

Section II **Student Number:**

90 marks

Allow about 2 hours and 45 minutes for this section

Answer each question in the spaces provided.

Your responses should include relevant mathematical reasoning and/or calculations.

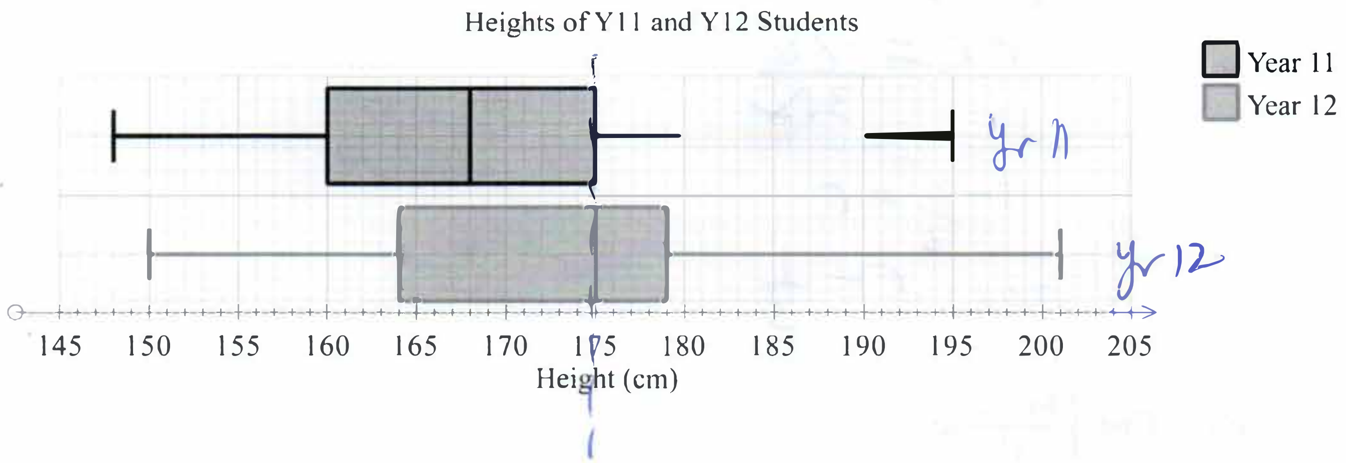
Extra writing space is provided at the end of each question. If you use this space, clearly indicate which question you are answering.

Question 11 (15 marks) **Marks**

(a) Find a primitive of $\frac{1}{5x+1}$. **2**

$\frac{1}{5}\log_e(5x+1)$

(b) The boxplot shows the heights of students in Year 11 and Year 12 at a school.



(i) What percentage of students in Year 11 have a height below 160 cm? **1**

25%

(ii) The number of students taller than 175 cm is coincidentally the same for both Year 11 and Year 12. If Year 11 has a total of 140 students, how many students are in Year 12? **1**

Yr 11: $0.25 \times 140 = 35$

Yr 12: $50\% = 35$

$35 \times 2 = 70$ students

Question 11 continues on page 10

Question 11 (continued)

(c) Find the common ratio of a geometric series with a first term of $\sqrt{2}$ and a limiting sum of $\frac{3\sqrt{2}}{2}$. **2**

$S_\infty = \frac{3\sqrt{2}}{2}$, $a = \sqrt{2}$, $r = ?$

$S_\infty = \frac{a}{1-r}$

$\frac{3\sqrt{2}}{2} = \frac{\sqrt{2}}{1-r}$

$3\sqrt{2}(1-r) = 2\sqrt{2}$

$1 - r = \frac{2\sqrt{2}}{3\sqrt{2}}$

$1 - \frac{2}{3} = r$

$\therefore r = \frac{1}{3}$

(d) Find $\int \frac{8x^3 - 3}{x^2}\, dx$ **2**

$\int \frac{8x^3}{x^2} - \frac{3}{x^2}\, dx$

$= \int 8x - 3x^{-2}\, dx$

$= \frac{8x^2}{2} - \frac{3x^{-1}}{-1} + C$

$= 4x^2 + \frac{3}{x} + C$

Question 11 continues on page 11

(e) A curve with the equation $y = f(x)$, has $\frac{dy}{dx} = x^3 + 2x - 7$.

(i) Find $\frac{d^2y}{dx^2}$ **1**

$\frac{d^2y}{dx^2} = 3x^2 + 2$

(ii) Show that $\frac{d^2y}{dx^2} \geq 2$ for all values of x. **2**

$x^2 \geq 0$

so $3x^2 \geq 0$

$3x^2 + 2 \geq 0 + 2$

$\therefore \frac{d^2y}{dx^2} \geq 2$ as required

(iii) The point $P(2, 4)$ lies on the curve. Find y in terms of x. **2**

$y = \frac{x^4}{4} + x^2 - 7x + C$

$4 = \frac{16}{4} + 4 - 14 + C$

$4 = 4 + 4 - 14 + C$

$4 = -6 + C$

$C = 10$

$\therefore y = \frac{x^4}{4} + x^2 - 7x + 10$

Question 11(e) continues on page 12

Question 11(e) (continued)

(iv) Find an equation for the normal to the curve at P, in the form $ax + by + c = 0$, where a, b and c are integers. **2**

$P(2,4)$ $m_T = 2^3 + 2\times2 - 7$

$= 8+4-7$

$= 5$

$m_T \times m_N = -1$ ($\perp$ lines)

$m_N = \frac{-1}{5}$

$y - y_1 = m(x - x_1)$

$y - 4 = \frac{-1}{5}(x-2)$

$5y - 20 = -x + 2$

$N:$ $x + 5y - 22 = 0$

End of Question 11

Question 12 (15 marks) **Student Number:**

(a) Let $h(x) = (x-2)(x^2+1)$.

(i) Find where the graph of $y = h(x)$ cuts the x-axis and y-axis. **2**

x-int ($y=0$) y-int ($x=0$)

$0 = (x-2)(x^2+1)$ $h(0) = (-2)(1)$

$x = 2$, $x^2 \neq -1$ $= -2$

$(2, 0)$ and $(0, -2)$

(ii) Find the coordinates of the stationary points on the curve with the equation $y = h(x)$ and determine their nature. **5**

$h'(x) = (x-2) \times 2x + (x^2+1)$

$= 2x^2 - 4x + x^2 + 1$

$h'(x) = 3x^2 - 4x + 1$

$h'(x) = 0$ (stationary pts)

$0 = 3x^2 - 4x + 1$

$0 = (3x-1)(x-1)$

$x = \frac{1}{3}, 1$

when $x = \frac{1}{3}$, $h\left(\frac{1}{3}\right) = \left(\frac{1}{3}-2\right)\left(\frac{1}{9}+1\right) = \frac{-50}{27}$ ($\doteqdot -1.85$)

when $x = 1$, $h(1) = (1-2)(1+1) = -2$

$h''(x) = 6x - 4$

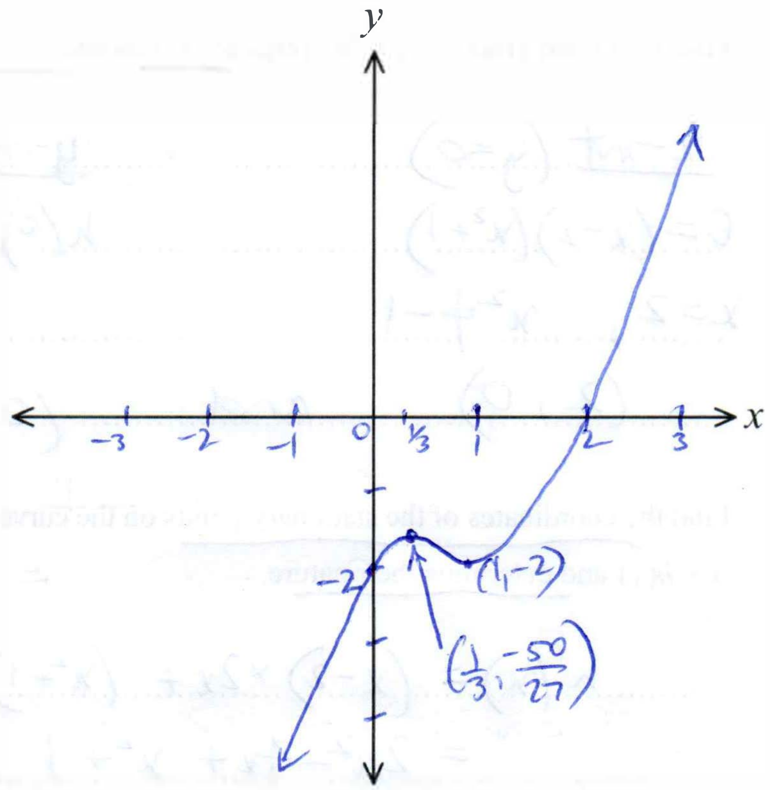
$h''\left(\frac{1}{3}\right) = 6 \times \frac{1}{3} - 4 = -2 < 0$ $\therefore$ maximum turning pt $\left(\frac{1}{3}, -\frac{50}{27}\right)$

$h''(1) = 6 - 4 = 2 > 0$ $\therefore$ minimum turning pt $(1, -2)$

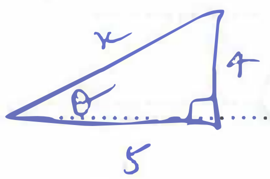
Question 12 continues on page 16

Question 12 (continued)

(iii) Sketch the graph of $y = h(x)$ showing stationary points and axes intercepts. **2**



(b) If $\tan\theta = \frac{4}{5}$, and θ is acute, find the exact value of $\sin\theta$. **2**



$x = \sqrt{5^2 + 4^2}$

$= \sqrt{25 + 16}$

$= \sqrt{41}$

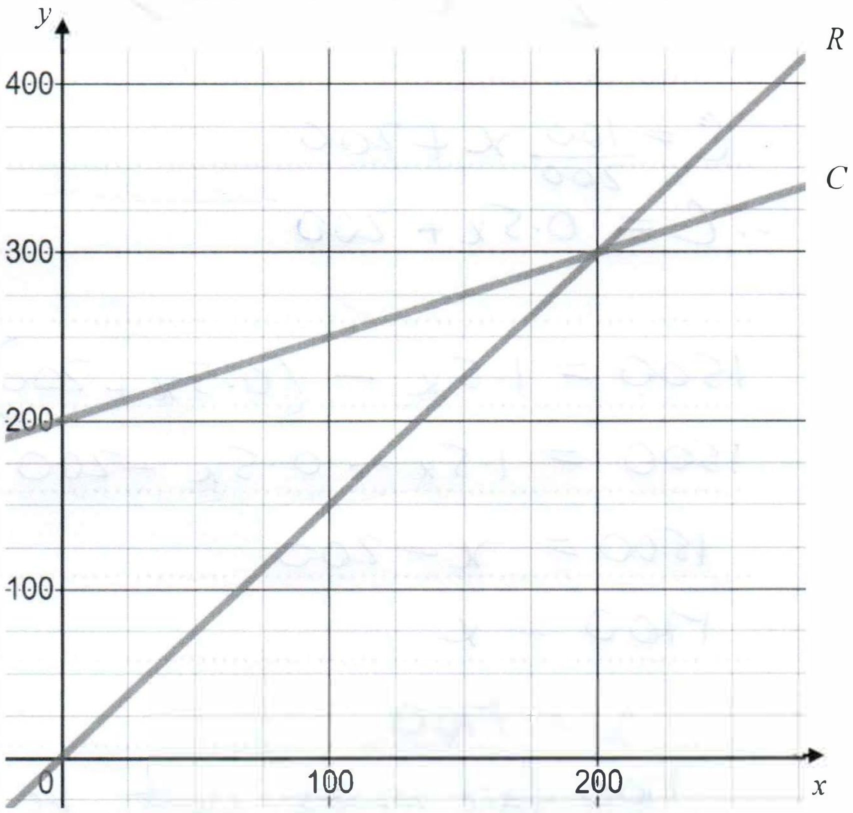
$\sin\theta = \frac{4}{\sqrt{41}}$ or $\frac{4\sqrt{41}}{41}$

Question 12 continues on page 17

Question 12 (continued)

(c) Terry is starting a small business making face masks.

Technology was used to draw straight-line graphs to represent the cost C, of Terry making face masks and the revenue, R from him selling them. The x-axis displays the number of face masks and the y-axis displays the cost/revenue in dollars.



(i) How many face masks must Terry sell to break even? **1**

200 face masks.

Question 12(c) continues on page 18

Question 12(c) (continued)

(ii) By first forming equations for cost C, and revenue R, determine how many face masks need to be sold to earn Terry a profit of \$1500. **3**

$R = \frac{300}{200} x$

$\therefore R = \frac{3}{2} x$ (or $R = 1.5x$)

$C = \frac{100}{200} x + 200$

$\therefore C = 0.5x + 200$

$1500 = 1.5x - (0.5x + 200)$

$1500 = 1.5x - 0.5x - 200$

$1500 = x - 200$

$1700 = x$

$x = 1700$

1700 face masks must be sold to earn a \$1500 profit.

End of Question 12

Question 13 (15 marks) **Student Number:**

(a) Find the exact value of $\int_0^{\frac{\pi}{6}} \sec^2 2x \, dx$ **2**

$\left[\frac{1}{2}\tan 2x\right]_0^{\pi/6} = \frac{1}{2}\tan\frac{\pi}{3} - \frac{1}{2}\tan 0$

$= \frac{1}{2} \times \sqrt{3}$

$= \frac{\sqrt{3}}{2}$

(b) Differentiate with respect to x.

(i) $y = \ln\left(3x^2 + 1\right)$ **1**

$y' = \frac{6x}{3x^2+1}$

(ii) $y = \frac{\sin x}{x^2}$ $\begin{matrix} u \\ v \end{matrix}$ **2**

$y' = \frac{vu' - uv'}{v^2}$

$u = \sin x$
$u' = \cos x$
$v = x^2$
$v' = 2x$

2mks this line → $= \frac{x^2\cos x - (\sin x) \times 2x}{(x^2)^2}$

$= \frac{x(x\cos x - 2\sin x)}{x^4}$

$\therefore y' = \frac{x\cos x - 2\sin x}{x^3}$

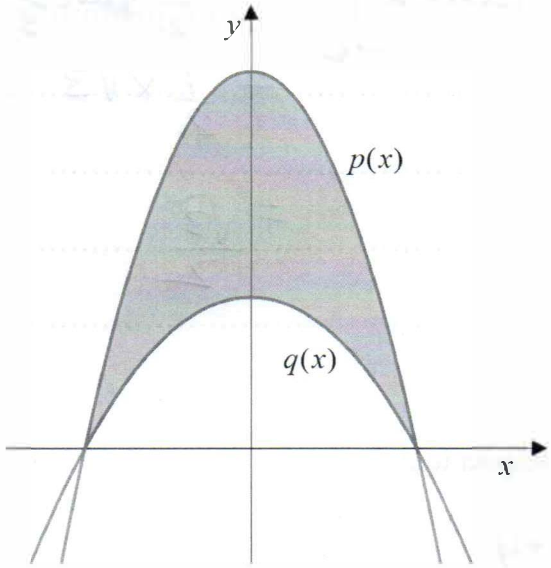
1mk even if quotient rule applied incorrectly after.

Question 13 continues on page 22

Question 13 (continued)

(c) Tess is creating a logo from the region intersecting the curves:

$$p(x) = (5-x)(5+x) \quad \text{and} \quad q(x) = \frac{2}{5}(5-x)(5+x).$$



(i) Show that the area A, of the shaded region is given by the expression **3**

$$A = \frac{6}{5}\int_0^5 25 - x^2\,dx.$$

$(5-x)(5+x) - \frac{2}{5}(5-x)(5+x)$

$= 25-x^2 - \frac{2}{5}(25-x^2)$

$= \frac{5(25-x^2) - 2(25-x^2)}{5}$

$= \frac{125-5x^2-50+2x^2}{5}$ 1

$= \frac{75-3x^2}{5} = \frac{3}{5}(25-x^2)$

These graphs intersect on x-axis $(y=0)$ 1

$\frac{3}{5}(25-x^2)=0$, $25-x^2=0$, $25=x^2$, $x=\pm 5$

and the graphs are symmetrical about the y-axis, so

$A = 2\times\frac{3}{5}\int_0^5 25-x^2\,dx = \frac{6}{5}\int_0^5 25-x^2\,dx$ as required 1

Question 13(c) continues on page 23

(ii) Hence, or otherwise, find the area of the shaded region. **2**

$A = \frac{6}{5}\left[25x - \frac{x^3}{3}\right]_0^5$

$\frac{6}{5}\left[25 \times 5 - \frac{125}{3} - (0 - 0)\right]$

$= \frac{6}{5}\left[\frac{125 \times 3 - 125}{3}\right]$

$= \frac{6}{15} \times 250$

$\therefore A = 100$ sq units

(d) For events A and B from a sample space, $P(A \mid B) = \frac{3}{4}$ and $P(B) = \frac{1}{7}$. **1**

Calculate $P(A \cap B)$.

$P(A/B) = \frac{P(A \cap B)}{P(B)}$

$\frac{3}{4} = \frac{P(A \cap B)}{\frac{1}{7}}$

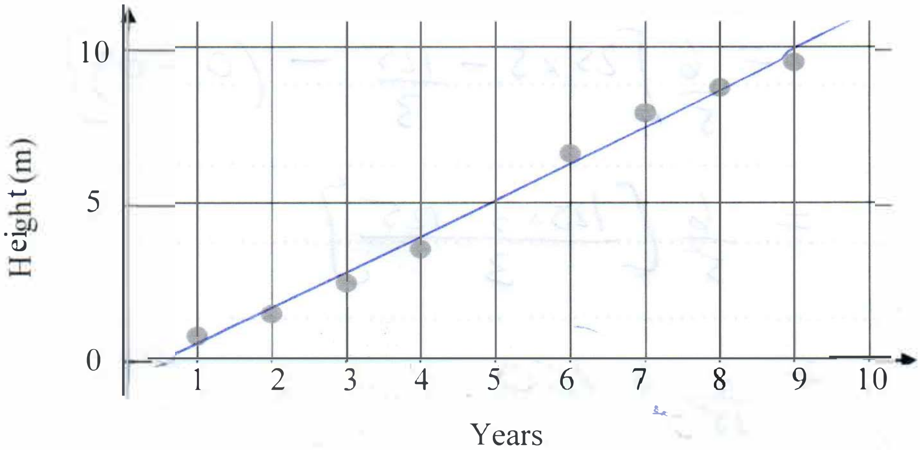
$P(A \cap B) = \frac{3}{4} \times \frac{1}{7}$

$\therefore P(A \cap B) = \frac{3}{28}$

Question 13 continues on page 24

(e) Charlotte is an agricultural scientist studying the growth of a particular tree over several years. The data she recorded is shown in the table and graph below.

Years since planting, t	1	2	3	4	6	7	8	9
Height of tree, H metres	0.7	1.4	2.4	3.5	6.6	7.9	8.7	9.5



(i) What is the correlation coefficient for this data (correct to 4 decimal places)? **1**

$r = 0.9952$

(ii) Find the equation of the least-squares line of best fit in terms of years (t) and height (H). Answer using values A and B correct to 2 decimal places, where $H = A + Bt$. **1**

$H = -0.8458\ldots + 1.1866t$

$\therefore H = -0.85 + 1.19t$

(iii) Use the equation to approximately determine how many years it will take for the tree to reach a height of 20 metres. Answer correct to 1 decimal place. **1**

[N.B. $t = 17.5$, $H = 19.975 < 20$]

When $H = 20$: $20 = 1.19t - 0.85$

$20.85 = 1.19t$

$t = \frac{20.85}{1.19} = 17.52$ (after 17.5 yrs)

(iv) What is the limitation of this model? **1**

As t increases its growth may not continue in a linear fashion.

End of Question 13

Question 14 (15 marks) **Student Number:**

(a) The probability that Chloe gets a concert booking with her band on any given weekend is 65%. What is the probability that she gets at least one booking over two consecutive weekends? **2**

1 − P(no booking 2 consec.) = 1 − 0·35 × 0·35
= 0·8775

(b) A circle is given by the equation $x^2 + y^2 - 4x + 6y = 12$.

Find the centre and radius of this circle. **3**

$x^2 - 4x + (-2)^2 + y^2 + 6y + 3^2 = 12 + 4 + 9$ 1

$(x-2)^2 + (y+3)^2 = 25$

centre (2, −3) 1

r = 5 units 1

Question 14 continues on page 28

Question 14 (continued)

(c) The score, X, for a biased spinner is given by the probability distribution:

x	2	4	6
$P(X = x)$	$\frac{1}{12}$	$\frac{2}{3}$	p

By finding the value of p, calculate the expected value and the variance of X. **3**

E(X) Var(X)

$E(X) = 2 \times \frac{1}{12} + 4 \times \frac{2}{3} + 6 \times \frac{1}{4}$

$= \frac{13}{3} = 4\frac{1}{3}$ 1

$E(X^2) = 2^2 \times \frac{1}{12} + 4^2 \times \frac{2}{3} + 6^2 \times \frac{1}{4} = 20$ 1

$Var(X) = E(X^2) - (E(X))^2$

$= 20 - (4\frac{1}{3})^2$

$= 20 - \frac{169}{9}$

$= \frac{180 - 169}{9}$

$= \frac{11}{9}$ ✓ 1

Question 14 continues on page 29

Question 14 (continued)

(d) The displacement of a particle is given by $x = t^2 - 4\log_e(t-1) + 5$, where x is in metres, t is in seconds and $t > 1$.

(i) Find the exact displacement of the particle when $t = 4$. **1**

$t=4$, $x = 4^2 - 4\log_e(4-1) + 5$

$= 16 - 4\ln 3 + 5$

$x = (21 - 4\ln 3)$ m

(ii) Find an expression for the particle's velocity and hence find when the particle comes to rest. **2**

$v = \frac{dx}{dt} = 2t - 4 \times \frac{1}{t-1} = 2t - \frac{4}{t-1}$

at rest, $v=0$ $\quad 0 = 2t - \frac{4}{t-1}$

$2t = \frac{4}{t-1}$

$2t(t-1) = 4$

$2t^2 - 2t - 4 = 0$

$2(t^2 - t - 2) = 0$

$(t-2)(t+1) = 0$

$t = 2, -1 \quad t > 0$

$\therefore t = 2$ seconds

(iii) Show that the acceleration remains positive for $t > 1$. **2**

$a = \frac{d^2x}{dt^2} = 2 + 4(t-1)^{-2}$

$= 2 + \frac{4}{(t-1)^2}$

$(t-1)^2 > 0$ for $t > 1$

$\therefore \frac{4}{(t-1)^2} > 0 \quad \therefore 2 + \frac{4}{(t-1)^2} > 0$ for $t > 1$

Question 14(d) continues on page 30

Question 14(d) (continued)

(iv) Find the exact distance travelled by the particle between the times the particle comes to rest and $t = 4$. **2**

$t=2$, $x = 2^2 - 4\log_e 1 + 5 = 4 - 0 + 5 = 9\text{m}$

$t=3$ $x = 3^2 - 4\ln(3^2-1) + 5$

$= (14 - 4\ln 2)$ m

$t=4$ $x = 21 - 4\log_e 3$ from (i)

Since $a > 0$ for $t > 1$

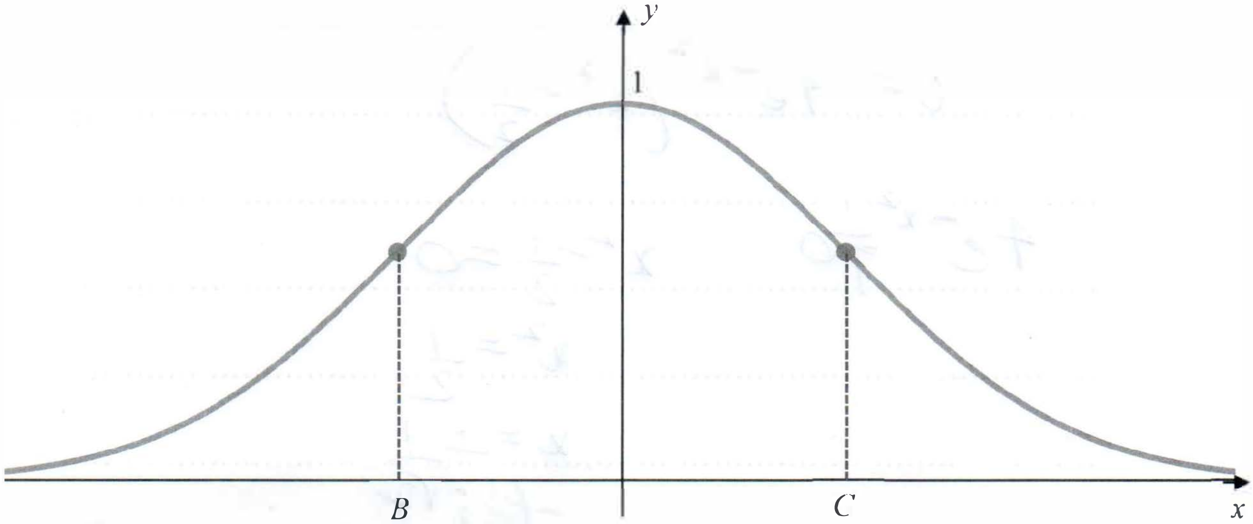
$21 - 4\ln 3 - 9 = (12 - 4\ln 3)$ m

End of Question 14

Question 15 (15 marks) **Student Number:**

(a) Isabelle is exploring the curve of the even function shown below, $y = e^{-x^2}$.

She knows there is a single stationary point shown at (0, 1) and two points of inflection are shown with x-values of B and C.



(i) Show that $\frac{d^2y}{dx^2} = 4e^{-x^2}\left(x^2 - \frac{1}{2}\right)$ **2**

$y = e^{-x^2}$

$\frac{dy}{dx} = -2xe^{-x^2}$

$\frac{d^2y}{dx^2} = -2x \times -2xe^{-x^2} + e^{-x^2} \times -2$

$= 4x^2e^{-x^2} - 2e^{-x^2}$

$= 4e^{-x^2}\left(x^2 - \frac{1}{2}\right)$ as required

Question 15(a) continues on page 34

(ii) Hence find the coordinates of the two points of inflection. **2**

$\frac{d^2y}{dx^2} = 0$ (inflexion pts)

$0 = 4e^{-x^2}\left(x^2 - \frac{1}{2}\right)$

$4e^{-x^2} \neq 0$ $\quad x^2 - \frac{1}{2} = 0$

$x^2 = \frac{1}{2}$

$x = \pm \frac{1}{\sqrt{2}}$

when $x = \frac{1}{\sqrt{2}}$, $y = e^{-\left(\frac{1}{\sqrt{2}}\right)^2}$ $\quad \therefore x = \pm\frac{1}{\sqrt{2}}$, $y = \frac{1}{\sqrt{e}}$

$= e^{-\frac{1}{2}}$

$= \frac{1}{\sqrt{e}}$

Isabelle wants to find the area of the region under the curve $y = e^{-x^2}$ bounded by the x-axis and the two inflection points, that is, $\int_B^C e^{-x^2}\,dx$.

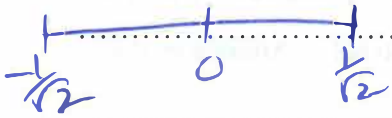
(iii) Explain how using a formula given on the Reference Sheet is unable to help provide an answer. **1**

Reference sheet: $\int f'(x)\,e^{f(x)}\,dx$ this integral is not in this format and so the rule cannot be applied.

Question 15(a) continues on page 35

(iv) Isabelle decides to approximate the area using the Trapezoidal Rule.

Show how she determined $\int_B^C e^{-x^2}\,dx \approx \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2e}}$ using three function values. **3**



$A \doteqdot \frac{1}{2\sqrt{2}}\left[f\left(-\frac{1}{\sqrt{2}}\right) + 2\times f(0) + f\left(\frac{1}{\sqrt{2}}\right)\right]$

$h = \frac{b-a}{n}$

$= \frac{1}{\sqrt{2}} \div 2$

$= \frac{1}{2\sqrt{2}}$

$= \frac{1}{2\sqrt{2}}\left[\frac{1}{\sqrt{e}} + 2\times 1 + \frac{1}{\sqrt{e}}\right]$

$= \frac{1}{2\sqrt{2}}\left[2 + \frac{2}{\sqrt{e}}\right]$

$f\left(-\frac{1}{\sqrt{2}}\right) = \frac{1}{\sqrt{e}}$

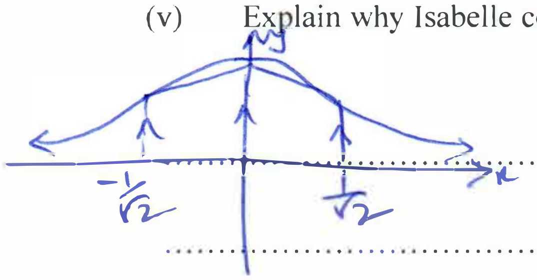
$= \frac{2}{2\sqrt{2}}\left[1 + \frac{1}{\sqrt{e}}\right]$

$f(0) = e^0 = 1$

$f\left(\frac{1}{\sqrt{2}}\right) = \frac{1}{\sqrt{e}}$

$= \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2e}}$ as required

(v) Explain why Isabelle correctly knows $\int_B^C e^{-x^2}\,dx > \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2e}}$. **1**



the trapezia are both below the curve $y = e^{-x^2}$ so the area of the trapezia is smaller than the area under the curve.

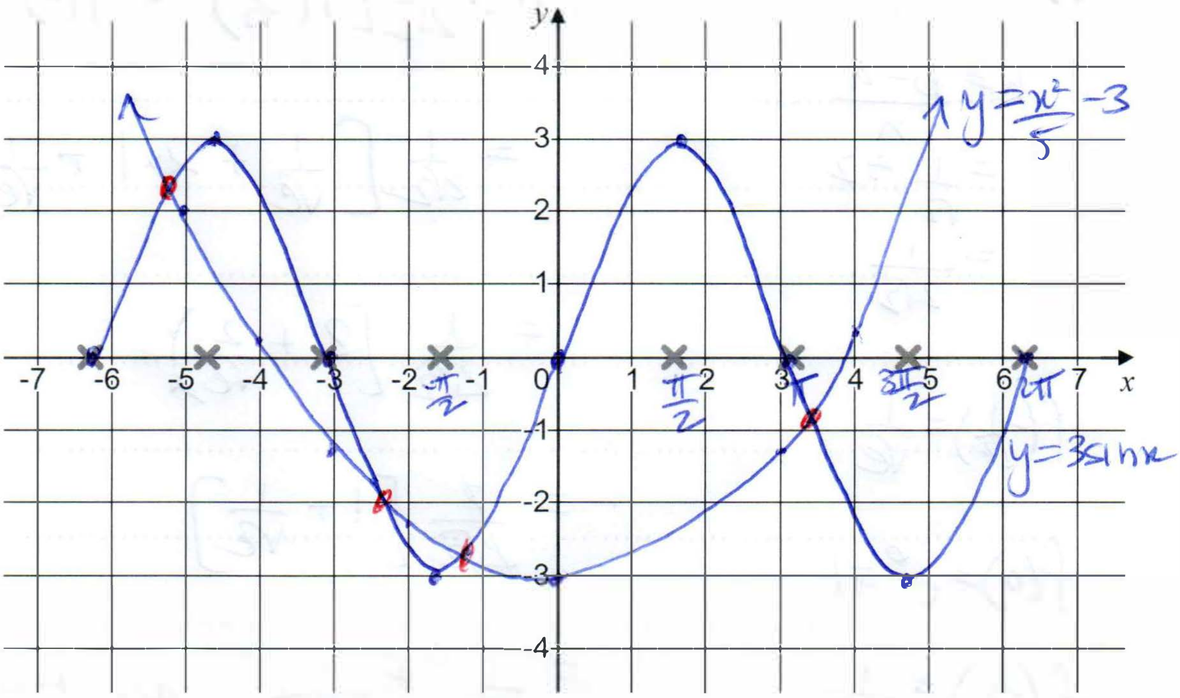
Question 15 continues on page 36

Question 15 (continued)

(b) Annie was preparing to determine how many solutions there are to the equation:

$$3\sin x = \frac{x^2}{5} - 3.$$

She plotted multiples of $\frac{\pi}{2}$ on the x-axis of the number plane below, shown by the crosses, to help. Draw graphs on this number plane to solve Annie's problem. **3**



The number of solutions: 4

Question 15 continues on page 37

Question 15 (continued)

(c) Show that $\sqrt{\dfrac{\sec\theta - 1}{\sec\theta + 1}} = \operatorname{cosec}\theta - \cot\theta$ **3**

$LHS = \sqrt{\dfrac{\sec\theta - 1}{\sec\theta + 1}}$

$= \sqrt{\dfrac{\frac{1}{\cos\theta} - 1}{\frac{1}{\cos\theta} + 1}}$

$= \sqrt{\dfrac{\frac{1-\cos\theta}{\cos\theta}}{\frac{1+\cos\theta}{\cos\theta}}}$

$= \sqrt{\dfrac{1-\cos\theta}{1+\cos\theta}}$

$= \sqrt{\dfrac{(1-\cos\theta)(1-\cos\theta)}{(1+\cos\theta)(1-\cos\theta)}}$

$= \sqrt{\dfrac{(1-\cos\theta)^2}{1-\cos^2\theta}}$

$= \dfrac{1-\cos\theta}{\sqrt{\sin^2\theta}}$

$= \dfrac{1-\cos\theta}{\sin\theta}$

$= \dfrac{1}{\sin\theta} - \dfrac{\cos\theta}{\sin\theta}$

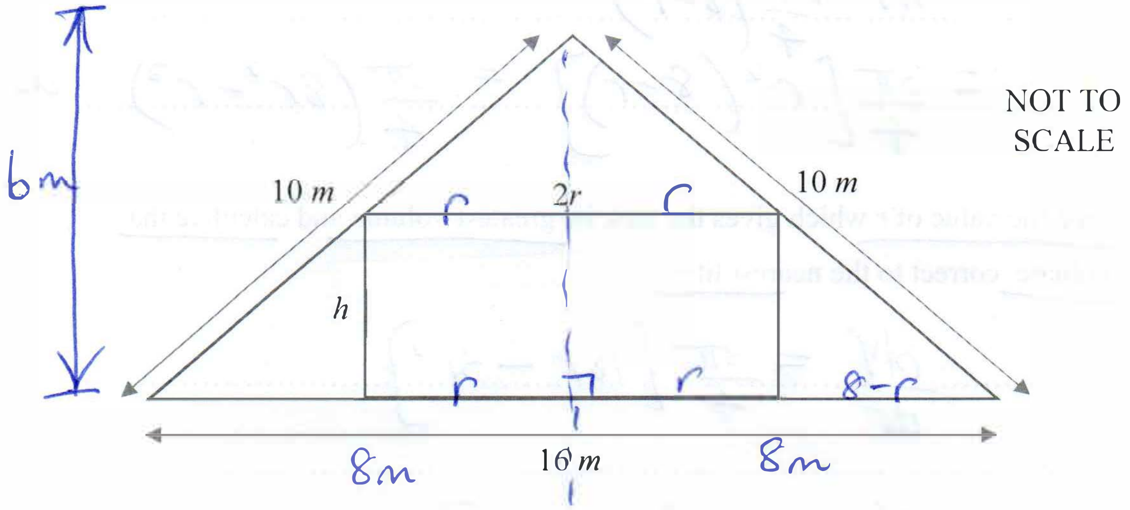
$= \operatorname{cosec}\theta - \cot\theta$

$= RHS.$

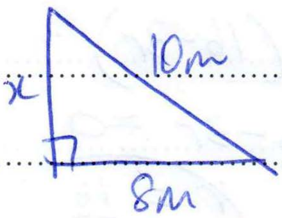
End of Question 15

Question 16 (15 marks) **Student Number:**

(a) In some rural areas hot water tanks are installed in the roofs of houses. The diagram below shows a cross-section of a cylindrical tank in a roof. The cylindrical tank snugly fits exactly into the roof with diameter $2r$ metres and height h metres.
The cross-section of the roof is an isosceles triangle with dimensions show.



(i) Show that the height of the roof is 6 metres. **1**



$x = \sqrt{10^2 - 8^2}$

$= \sqrt{36}$

$x = 6m$

(ii) Hence show that $h = \frac{3}{4}(8 - r)$. **2**

$\frac{8}{8-r} = \frac{6}{h}$ (using Similar Δ's)

$8h = 6(8-r)$

$h = \frac{6}{8}(8-r)$

$\therefore h = \frac{3}{4}(8-r)$

Question 16(a) continues on page 40

Question 16(a) (continued)

(iii) Show that the volume of the cylindrical tank can be expressed by **1**

$$V = \frac{3\pi}{4}\left(8r^2 - r^3\right).$$

$V = \pi r^2 h$

$= \pi r^2 \times \frac{3}{4}(8-r)$

$= \frac{3\pi}{4}\left[r^2(8-r)\right] = \frac{3\pi}{4}\left(8r^2 - r^3\right)$ as req'd

(iv) Find the value of r which gives the tank its greatest volume and calculate that volume, correct to the nearest litre. **4**

$\frac{dV}{dr} = \frac{3\pi}{4}\left[16r - 3r^2\right]$

$\frac{d^2V}{dr^2} = \frac{3\pi}{4}\left[16 - 6r\right]$

$\frac{dV}{dr} = 0 \quad 0 = \frac{3\pi}{4} r(16-3r)$

$r = 0, \quad 16 - 3r = 0$

$r = \frac{16}{3}, \quad r > 0$

when $r = \frac{16}{3}$ $\frac{d^2V}{dr^2} = \frac{3\pi}{4}\left(16 - 6 \times \frac{16}{3}\right)$

$= \frac{3\pi}{4}(16-32) = \frac{3\pi}{4} \times -16 < 0$

$\therefore$ maximum (greatest) volume

when $r = \frac{16}{3}$ $V = \frac{3\pi}{4}\left(8 \times \left(\frac{16}{3}\right)^2 - \left(\frac{16}{3}\right)^3\right)$

$= \frac{3\pi}{4} \times \frac{2048}{27}$ 512 $= \frac{512\pi}{9}$

$= 178.7217154$

$\times 1000$

$= 178722$ L

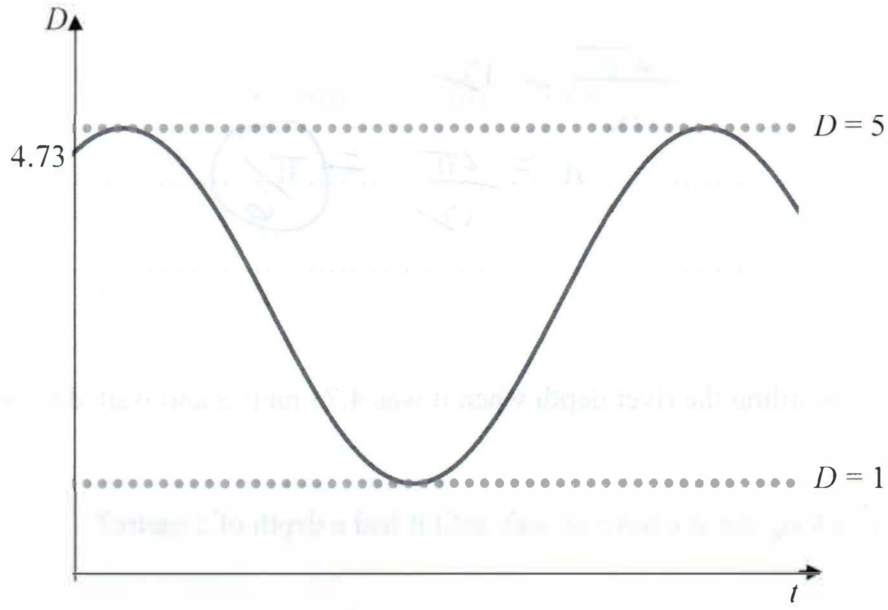
or 178.722 kL

$1\text{m}^3 = 1000$ L

$1\text{m}^3 = 1$ kL

Question 16 continues on page 41

(b)



Sophie has developed an equation, drawn above, for the depth D, of a river near her home. The depth is modelled by the function:

$$D = a\sin\left(nt + \frac{\pi}{3}\right) + c$$

where D is measured in metres and t is the time in hours. The time between successive peaks in Sophie's model is exactly 12 hours.

(i) What is the value of the amplitude, a? **1**

$5 - 1 = 4$

$4 \div 2 = 2$ $a = 2$

(ii) Find the value of c. **1**

$c = 3$

Question 16(b) continues on page 42

Question 16(b) (continued)

(iii) Find the value of n. **1**

$\frac{2\pi}{n} = 12$

$n = \frac{2\pi}{12} = \frac{\pi}{6}$

Sophie started recording the river depth when it was 4.73 metres and waited to cross it safely.

(iv) How long did she have to wait until it had a depth of 1 metre? **2**

$1 = 2\sin\left(\frac{\pi}{6}t + \frac{\pi}{3}\right) + 3$

$-2 = 2\sin\left(\frac{\pi}{6}t + \frac{\pi}{3}\right)$

$-1 = \sin\left(\frac{\pi}{6}t + \frac{\pi}{3}\right)$ $\sin\frac{3\pi}{2} = -1$

$\frac{\pi}{6}t + \frac{\pi}{3} = \frac{3\pi}{2}$

$\frac{\pi t}{6} = \frac{3\pi}{2} - \frac{\pi}{3}$

$\frac{\pi}{6}t = \frac{9\pi - 2\pi}{6}$

$t = 9 - 2$

$\therefore t = 7$ hrs

. **Question 16(b) continues on page 43**

Question 16(b) (continued)

(v) From her record, when was the greatest rate of drop in depth and what was that rate at this time? Answer correct to 2 decimal places. **2**

$t=1$ to $t=7$ 6 h

$6 \div 2 = 3$

$t = 1+3 = 4$ for greatest rate of drop in depth

At $t=4$, $\frac{dD}{dt} = \left(\frac{\pi}{6} \times 2\right)\cos\left(\frac{\pi}{6} \times 4 + \frac{\pi}{3}\right)$

$= \frac{\pi}{3} \cos\left(\frac{3\pi}{3}\right)$

$= \frac{\pi}{3} \cos \pi$

$= \pi/3 \times -1$

$= -\pi/3$ m/h

$= -1.0471...$

$\frac{dD}{dt} = -1.05$ m/h (2 dp)

END OF PAPER